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FILM FLOW OF A NON-NEWTONIAN LIQUID OVER ROTATING SURFACES

K. D. Vachagin, N. Kh. Zinnatulin, and N. V. Tyabin

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An account is given of the derivation of the differential equations of motion of a non-Newtonian liquid and of heat transfer in an orthogonal curvilinear system of coordinates. The flow of this liquid in the form of a film over a rotating disc of arbitrary shape is examined.

The internal friction of many non-Newtonian liquids in uniform flow is well described by the power law [1-3]:

$$\tau = K\dot{\gamma}^n. \tag{1}$$

If the concept of effective viscosity [1] is used, (1) may be written as

$$\tau = \mu_{\text{eff}}\,\dot{\gamma}, \tag{2}$$

where

$$\mu_{\text{eff}} = K\dot{\gamma}^{n-1}.$$

By analogy with Newton's general hypothesis for a viscous liquid [4], the relation between stresses and rates of strain may be written for an incompressible non-Newtonian liquid in the form

$$D_\sigma = 2\mu^*_{\text{eff}} D_\varepsilon. \tag{3}$$

It follows from (3) that

$$T = \mu^*_{\text{eff}} E. \tag{4}$$

The possibility of this kind of generalization was pointed out in [5], and the validity of a relation of type (4) was shown experimentally in [6].

Generalizing (1) to the case of three-dimensional flow, we have

$$T = KE^n. \tag{5}$$

The analogous law of internal friction for plastic bodies was obtained in [7].

From (4) and (5) we obtain

$$\mu^*_{\text{eff}} = KE^{n-1}. \tag{6}$$

On the basis of (3) and (6) and the differential equations of motion of a continuous medium [4], we may obtain the differential equations of motion in velocity components describing the general case of flow of a non-Newtonian liquid in an orthogonal curvilinear system of coordinates q_1, q_2, q_3.

For this it is necessary to have a relation between the components of stress and rate of strain.

From (3) and (6) we obtain

$$\begin{aligned}
p_{11} &= -p + 2KE^{n-1}\left(\frac{1}{H_1}\frac{\partial v_1}{\partial q_1} + \frac{v_2}{H_1H_2}\frac{\partial H_1}{\partial q_2} + \frac{v_3}{H_1H_3}\frac{\partial H_1}{\partial q_3}\right),\\
p_{12} &= KE^{n-1}\left[\frac{H_2}{H_1}\frac{\partial}{\partial q_1}\left(\frac{v_2}{H_2}\right) + \frac{H_1}{H_2}\frac{\partial}{\partial q_2}\left(\frac{v_1}{H_1}\right)\right],\\
p_{22} &= -p + 2KE^{n-1}\left(\frac{1}{H_2}\frac{\partial v_2}{\partial q_2} + \frac{v_3}{H_2H_3}\frac{\partial H_2}{\partial q_3} + \frac{v_1}{H_1H_2}\frac{\partial H_2}{\partial q_1}\right),\\
p_{13} &= KE^{n-1}\left[\frac{H_3}{H_1}\frac{\partial}{\partial q_1}\left(\frac{v_3}{H_3}\right) + \frac{H_1}{H_3}\frac{\partial}{\partial q_3}\left(\frac{v_1}{H_1}\right)\right],\\
p_{33} &= -p + 2KE^{n-1}\left(\frac{1}{H_3}\frac{\partial v_3}{\partial q_3} + \frac{v_1}{H_1H_3}\frac{\partial H_3}{\partial q_1} + \frac{v_2}{H_2H_3}\frac{\partial H_3}{\partial q_2}\right),
\end{aligned} \tag{7}$$

$$p_{23} = KE^{n-1}\left[\frac{H_3}{H_2}\frac{\partial}{\partial q_2}\left(\frac{v_3}{H_3}\right)+\frac{H_2}{H_3}\frac{\partial}{\partial q_3}\left(\frac{v_2}{H_2}\right)\right]. \qquad (7) \text{ (cont'd)}$$

Substituting (7) into the equations of motion of a continuous medium in stress components, we have

$$\frac{\partial v_1}{\partial t}+\frac{v_1}{H_1}\frac{\partial v_1}{\partial q_1}+\frac{v_2}{H_2}\frac{\partial v_1}{\partial q_2}+\frac{v_3}{H_3}\frac{\partial v_1}{\partial q_3}+\frac{v_1v_2}{H_1H_2}\frac{\partial H_1}{\partial q_2}+$$
$$+\frac{v_1v_3}{H_1H_3}\frac{\partial H_1}{\partial q_3}-\frac{v_2^2}{H_1H_2}\frac{\partial H_2}{\partial q_1}-\frac{v_3^2}{H_3H_1}\frac{\partial H_3}{\partial q_1}=$$
$$=F_1-\frac{1}{\rho H_1}\frac{\partial p}{\partial q_1}+\frac{K}{\rho}E^{n-1}M_1+\frac{2K}{\rho H_1}\times$$
$$\times\left(\frac{1}{H_1}\frac{\partial v_1}{\partial q_1}+\frac{v_2}{H_1H_2}\frac{\partial H_1}{\partial q_2}+\frac{v_3}{H_1H_3}\frac{\partial H_1}{\partial q_3}\right)\times$$
$$\times\frac{\partial(E^{n-1})}{\partial q_1}+\frac{K}{\rho H_2}\left[\frac{H_2}{H_1}\frac{\partial}{\partial q_1}\left(\frac{v_2}{H_2}\right)+\right.$$
$$\left.+\frac{H_1}{H_2}\frac{\partial}{\partial q_2}\left(\frac{v_1}{H_1}\right)\right]\frac{\partial(E^{n-1})}{\partial q_2}+$$
$$+\frac{K}{\rho H_3}\left[\frac{H_3}{H_1}\frac{\partial}{\partial q_1}\left(\frac{v_3}{H_3}\right)+\frac{H_1}{H_3}\frac{\partial}{\partial q_3}\left(\frac{v_1}{H_1}\right)\right]\frac{\partial(E^{n-1})}{\partial q_3},$$

$$\frac{\partial v_2}{\partial t}+\frac{v_2}{H_2}\frac{\partial v_2}{\partial q_2}+\frac{v_3}{H_3}\frac{\partial v_2}{\partial q_3}+\frac{v_1}{H_1}\frac{\partial v_2}{\partial q_1}+\frac{v_2v_3}{H_2H_3}\frac{\partial H_2}{\partial q_3}+$$
$$+\frac{v_2v_1}{H_2H_1}\frac{\partial H_2}{\partial q_1}-\frac{v_3^2}{H_2H_3}\frac{\partial H_3}{\partial q_2}-\frac{v_1^2}{H_1H_2}\frac{\partial H_1}{\partial q_2}=F_2-$$
$$-\frac{1}{\rho H_2}\frac{\partial p}{\partial q_2}+\frac{K}{\rho}E^{n-1}M_2+\frac{2K}{\rho H_2}\times \qquad (8)$$
$$\times\left(\frac{1}{H_2}\frac{\partial v_2}{\partial q_2}+\frac{v_3}{H_2H_3}\frac{\partial H_2}{\partial q_3}+\frac{v_1}{H_1H_2}\frac{\partial H_2}{\partial q_1}\right)\times$$
$$\times\frac{\partial(E^{n-1})}{\partial q_2}+\frac{K}{\rho H_3}\left[\frac{H_3}{H_2}\frac{\partial}{\partial q_2}\left(\frac{v_3}{H_3}\right)+\right.$$
$$\left.+\frac{H_2}{H_3}\frac{\partial}{\partial q_3}\left(\frac{v_2}{H_2}\right)\right]\frac{\partial(E^{n-1})}{\partial q_3}+\frac{K}{\rho H_1}\times$$
$$\times\left[\frac{H_2}{H_1}\frac{\partial}{\partial q_1}\left(\frac{v_2}{H_2}\right)+\frac{H_1}{H_2}\frac{\partial}{\partial q_2}\left(\frac{v_1}{H_1}\right)\right]\frac{\partial(E^{n-1})}{\partial q_1},$$

$$\frac{\partial v_3}{\partial t}+\frac{v_3}{H_3}\frac{\partial v_3}{\partial q_3}+\frac{v_1}{H_1}\frac{\partial v_3}{\partial q_1}+\frac{v_2}{H_2}\frac{\partial v_3}{\partial q_2}+\frac{v_3v_1}{H_3H_1}\frac{\partial H_3}{\partial q_1}+$$
$$+\frac{v_3v_2}{H_3H_2}\frac{\partial H_3}{\partial q_2}-\frac{v_1^2}{H_3H_1}\frac{\partial H_1}{\partial q_3}-\frac{v_2^2}{H_2H_3}\frac{\partial H_2}{\partial q_3}=F_3-$$
$$-\frac{1}{\rho H_3}\frac{\partial p}{\partial q_3}+\frac{K}{\rho}E^{n-1}M_3+\frac{2K}{\rho H_3}\times$$
$$\times\left(\frac{1}{H_3}\frac{\partial v_3}{\partial q_3}+\frac{v_1}{H_1H_3}\frac{\partial H_3}{\partial q_1}+\frac{v_2}{H_2H_3}\frac{\partial H_3}{\partial q_2}\right)\frac{\partial(E^{n-1})}{\partial q_3}+\frac{K}{\rho H_1}\times$$
$$\times\left[\frac{H_3}{H_1}\frac{\partial}{\partial q_1}\left(\frac{v_3}{H_3}\right)+\frac{H_1}{H_3}\frac{\partial}{\partial q_3}\left(\frac{v_1}{H_1}\right)\right]\frac{\partial(E^{n-1})}{\partial q_1}+$$
$$+\frac{K}{\rho H_2}\left[\frac{H_3}{H_2}\frac{\partial}{\partial q_2}\left(\frac{v_3}{H_3}\right)+\frac{H_2}{H_3}\frac{\partial}{\partial q_3}\left(\frac{v_2}{H_2}\right)\right]\frac{\partial(E^{n-1})}{\partial q_2},$$

where

$$E = \left\{ 2\left(\frac{1}{H_1} \frac{\partial v_1}{\partial q_1} + \frac{v_2}{H_1 H_2} \frac{\partial H_1}{\partial q_2} + \frac{v_3}{H_1 H_3} \frac{\partial H_1}{\partial q_3} \right)^2 + \right.$$
$$+ 2\left(\frac{1}{H_2} \frac{\partial v_2}{\partial q_2} + \frac{v_3}{H_2 H_3} \frac{\partial H_2}{\partial q_3} + \frac{v_1}{H_1 H_2} \frac{\partial H_2}{\partial q_1} \right)^2 +$$
$$+ 2\left(\frac{1}{H_3} \frac{\partial v_3}{\partial q_3} + \frac{v_1}{H_1 H_3} \frac{\partial H_3}{\partial q_1} + \frac{v_2}{H_2 H_3} \frac{\partial H_3}{\partial q_2} \right)^2 +$$
$$+ \left[\frac{H_2}{H_1} \frac{\partial}{\partial q_1} \left(\frac{v_2}{H_2} \right) + \frac{H_1}{H_2} \frac{\partial}{\partial q_2} \left(\frac{v_1}{H_1} \right) \right]^2 +$$
$$+ \left[\frac{H_3}{H_2} \frac{\partial}{\partial q_2} \left(\frac{v_3}{H_3} \right) + \frac{H_2}{H_3} \frac{\partial}{\partial q_3} \left(\frac{v_2}{H_2} \right) \right]^2 +$$
$$\left. + \left[\frac{H_3}{H_1} \frac{\partial}{\partial q_1} \left(\frac{v_3}{H_3} \right) + \frac{H_1}{H_3} \frac{\partial}{\partial q_3} \left(\frac{v_1}{H_1} \right) \right]^2 \right\}^{1/2},$$

$$M_i = M\left(H_i, \frac{\partial H_i}{\partial q_i}, v_i, \frac{\partial v_i}{\partial q_i}, \frac{\partial^2 v_i}{\partial q_i^2} \right), \quad i = 1, 2, 3.$$

Relations for M_1, M_2, M_3 were given in [8].

It is necessary to add to the set of equations (8) the equation of continuity [4]:

$$\frac{\partial}{\partial q_1} (v_1 H_2 H_3) + \frac{\partial}{\partial q_2} (v_2 H_3 H_1) + \frac{\partial}{\partial q_3} (v_3 H_1 H_2) = 0. \tag{9}$$

Equations (8) and (9) describe the general case of flow of a non-Newtonian liquid in arbitrary orthogonal curvilinear coordinates. From these, in particular, we may obtain the equations of motion of a non-Newtonian liquid in rectangular, cylindrical, and spherical coordinates. For this, it is only necessary to have the appropriate Lamé coefficients for these coordinates.

A laminar flow regime [1] is characteristic of the processing of polymer materials. In this case, the heat flux is the sum of the heat due to conduction and that due to dissipation of mechanical energy.

If it is assumed that the specific heat and the thermal conductivity are independent of temperature, the heat flux equation for non-Newtonian liquids, as for Newtonian liquids, has the form [4]

$$\rho \frac{c_p}{A} \left(\frac{\partial T^0}{\partial T} + \frac{v_1}{H_1} \frac{\partial T^0}{\partial q_1} + \frac{v_2}{H_2} \frac{\partial T^0}{\partial q_2} + \frac{v_3}{H_3} \frac{\partial T^0}{\partial q_3} \right) =$$
$$= D + \frac{\lambda}{A H_1 H_2 H_3} \left[\frac{\partial}{\partial q_1} \left(\frac{H_2 H_3}{H_1} \frac{\partial T^0}{\partial q_1} \right) + \right. \tag{10}$$
$$\left. + \frac{\partial}{\partial q_2} \left(\frac{H_1 H_3}{H_2} \frac{\partial T^0}{\partial q_2} \right) + \frac{\partial}{\partial q_3} \left(\frac{H_1 H_2}{H_3} \frac{\partial T^0}{\partial q_3} \right) \right].$$

For non-Newtonian liquids D will be given by

$$D = 2KE^{n-1} \left\{ \left(\frac{1}{H_1} \frac{\partial v_1}{\partial q_1} + \frac{v_2}{H_1 H_2} \frac{\partial H_1}{\partial q_2} + \frac{v_3}{H_1 H_3} \frac{\partial H_1}{\partial q_3} \right)^2 + \right.$$
$$+ \left(\frac{1}{H_2} \frac{\partial v_2}{\partial q_2} + \frac{v_1}{H_2 H_3} \frac{\partial H_2}{\partial q_3} + \frac{v_1}{H_1 H_2} \frac{\partial H_2}{\partial q_1} \right)^2 + \tag{11}$$
$$+ \left(\frac{1}{H_3} \frac{\partial v_3}{\partial q_3} + \frac{v_1}{H_1 H_3} \frac{\partial H_3}{\partial q_1} + \frac{v_2}{H_2 H_3} \frac{\partial H_3}{\partial q_2} \right)^2 +$$
$$+ \left[\frac{H_2}{H_1} \frac{\partial}{\partial q_1} \left(\frac{v_2}{H_2} \right) + \frac{H_1}{H_2} \frac{\partial}{\partial q_2} \left(\frac{v_1}{H_1} \right) \right]^2 +$$
$$+ \left[\frac{H_3}{H_1} \frac{\partial}{\partial q_1} \left(\frac{v_3}{H_3} \right) + \frac{H_1}{H_3} \frac{\partial}{\partial q_3} \left(\frac{v_1}{H_1} \right) \right]^2 +$$

$$+\left[\frac{H_3}{H_2}\frac{\partial}{\partial q_2}\left(\frac{v_3}{H_3}\right)+\frac{H_2}{H_3}\frac{\partial}{\partial q_3}\left(\frac{v_2}{H_2}\right)\right]^2\Bigg\}. \quad (11) \text{ (cont'd)}$$

It is not difficult to obtain from (10) and (11) the heat flux equation for a non-Newtonian liquid in rectangular, cylindrical, and spherical coordinates.

We shall examine steady flow of a non-Newtonian incompressible homogeneous liquid, flowing over a rotating surface as a thin continuous laminar film. We shall use a special set of coordinates r, φ, δ.

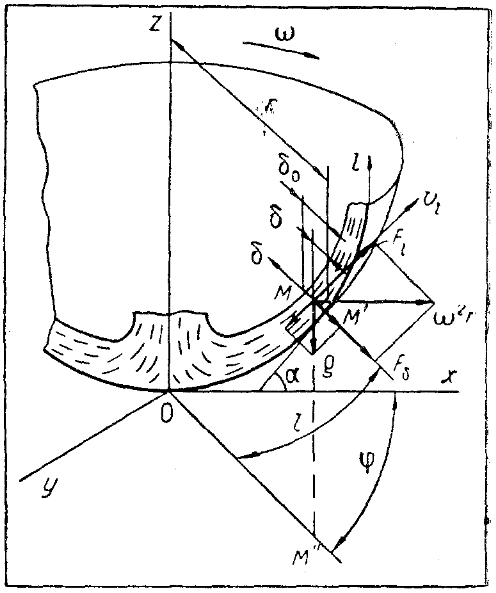

Fig. 1. Diagram of liquid flow over surface.

In the solution we shall assume the following conditions: 1) the surface is given by the equation $(z = f(\sqrt{x^2+y^2})$; 2) the angular velocity of rotation of the liquid is equal to the rate of rotation of the surface; 3) the liquid film thickness δ_0 is considerably less than the corresponding coordinates l, r; 4) the flow is symmetrical about the axis of rotation of the surface; 5) the influence of friction forces between the film and the surrounding medium and of surface tension forces on the flow is insignificant; 6) the relative velocity of motion of the liquid film is considerably less than the circular velocity; 7) the radius of curvature of the surface is much greater than the film thickness; 8) the system of coordinates l, φ, δ is rigidly tied to the rotating surface.

From examination of Fig. 1 we may write

$$x' = (r - \delta\sin\alpha)\cos\varphi,\ y' = -(r-\delta\sin\alpha)\sin\varphi,\ z' = = z+\delta\cos\alpha.$$

Here

$$\sin\alpha = \frac{dz}{dr}\Big/\sqrt{1+\left(\frac{dz}{dr}\right)^2},\quad \cos\alpha = 1\Big/\sqrt{1+\left(\frac{dz}{dr}\right)^2}.$$

The Lamé coefficients for this case, allowing for condition 3, are: $H_l = 1,\ H_\varphi = r,\ H_\delta = 1$. Then the equations of motion of a non-Newtonian liquid (7)-(8), allowing for the above conditions, have the form

$$K\frac{\partial}{\partial\delta}\left[\left|\frac{\partial v_l}{\partial\delta}\right|^{n-1}\frac{\partial v_l}{\partial\delta}\right]-\frac{\partial p}{\partial l}+\rho F_l = 0, \quad (12)$$

$$\frac{\partial p}{\partial\varphi} = 0, \quad (13)$$

$$-\frac{\partial p}{\partial\delta}+\rho F_\delta = 0. \quad (14)$$

The quantities F_l and F_δ are given by

$$F_l = \omega^2 r\cos\alpha - g\sin\alpha,\quad F_\delta = -(\omega^2 r\sin\alpha + g\cos\alpha).$$

Integration of (14) gives

$$p = \rho F_\delta\delta + C_1(l). \quad (15)$$

To determine $C_1(l)$, we use the condition that when $\delta = \delta_0$, $p = p_0$. Thus,

$$p = -(\delta_0-\delta)\rho F_\delta + p_0. \quad (16)$$

From (16), knowing the equation of the surface, we can find the pressure at any point in a film of non-Newtonian liquid flowing over a rotating surface.

Assuming that the mean pressure over the film thickness is given by

$$p_m = \delta_0^{-1} \int_{\delta}^{\delta_0} p d\delta,$$

from (16), we have

$$p_m = -\frac{\delta_0}{1} \rho F_\delta + p_0. \tag{17}$$

The relation for the velocity may be obtained by integrating (12), but, before integrating, we shall determine the order of importance of its terms. Differentiating (16) with respect to l, we obtain

$$\frac{dp}{dl} = \rho F_\delta \frac{d\delta_0}{dl} - (\delta_0 - \delta) \rho \frac{dF_\delta}{dl}.$$

If $\operatorname{ctg}\alpha \gg \delta_0/l$, then $\frac{dp}{dl} \sim \rho\omega^2 r \delta_0/l$, $\rho F_l \sim \rho\omega^2 r \operatorname{ctg}\alpha$. Therefore dp/dl may be discarded. We shall estimate the value of angle α when the condition $\operatorname{ctg}\alpha \gg \delta_0/l$ holds. It is known from experimental investigations that the film thickness may range from some tens of microns to several millimeters, and the length of the surface generator from several tens to several hundreds of millimeters. Therefore for ordinary cases $\delta_0/l \sim 10^{-2}$. Then the greatest value of α may be in the range 75–80°.

A single integration of (12) without account for dp/dl gives

$$K\left[\left|\frac{\partial v_l}{\partial \delta}\right|^{n-1} \frac{\partial v_l}{\partial \delta}\right] = -\rho F_l \delta + C_2(l).$$

$C_2(l)$ is found from the condition of no friction on the film surface, i.e., when $\delta = \delta_0$ $\partial v_l/\partial\delta = 0$. Then $C_2(l) = \rho F_l \delta_0$. Allowing for the value of $C_2(l)$, we find

$$\frac{\partial v_l}{\partial \delta} = [(\delta_0 - \delta)\frac{\rho}{K} F_l]^{1/n}.$$

Further integration of this equation gives

$$v_l = -\left(\frac{\rho}{K} F_l\right)^{\frac{1}{n}} \left(\frac{n}{1+n}\right) (\delta_0 - \delta)^{\frac{1+n}{n}} + C_3(l).$$

We find the value of $C_3(l)$ from the condition of adhesion of the liquid to the surface, i.e., when $\delta = 0$ $v_l = 0$. Thus, for v_l we obtain

$$v_l = \left[\delta_0^{\frac{1+n}{n}} - (\delta_0 - \delta)^{\frac{1+n}{n}}\right] \frac{n}{1+n} (\rho F_l/K)^{\frac{1}{n}}. \tag{18}$$

The mean value of the meridional velocity over the film thickness may be determined from

$$v_{l\,m} = \frac{1}{\delta_0} \int_0^{\delta_0} v_l d\delta = \delta_0^{\frac{1+n}{n}} \left(\frac{n}{1+2n}\right) \left(\frac{\rho}{K} F_l\right)^{\frac{1}{n}}. \tag{19}$$

Having an expression for v_{lm}, from the equation of constancy of mass flow, $q = 2\pi r \delta_0 v_{lm}$, it is not difficult to obtain a relation for the film thickness

$$\delta_0 = \left[\frac{q}{2\pi r}\left(\frac{1+2n}{n}\right)\left(\frac{K}{\rho F_l}\right)^{\frac{1}{n}}\right]^{\frac{n}{1+2n}}. \tag{20}$$

From (20), for a known F_l, we can calculate the thickness of a film of non-Newtonian liquid flowing over a rotating surface.

Let us examine the flow of a liquid over some actual surfaces.

Curvilinear diffuser. The equation of the surface is given:

$$z = Br^a, \tag{21}$$

where $B > 0,\ a > 0.$

We consider that the liquid moves over the inside surface. Then we have

$$F_l = \frac{\omega^2 r - Bagr^{a-1}}{\sqrt{1 + (Bar^{a-1})^2}},\quad F_\delta = -\frac{Ba\,\omega^2 r^a + g}{\sqrt{1 + (Bar^{a-1})^2}}.$$

In the special case when $B = \operatorname{ctg}\beta,\ a = 1,$ Eq. (21) is the equation of a conical surface.

Spherical surface. Let the liquid flow over the inside surface of a rotating sphere. The origin of coordinates is located on the surface and on the axis of rotation. Then the equation of the spherical surface may be written as

$$z = -\sqrt{R^2 + r^2} + R. \tag{22}$$

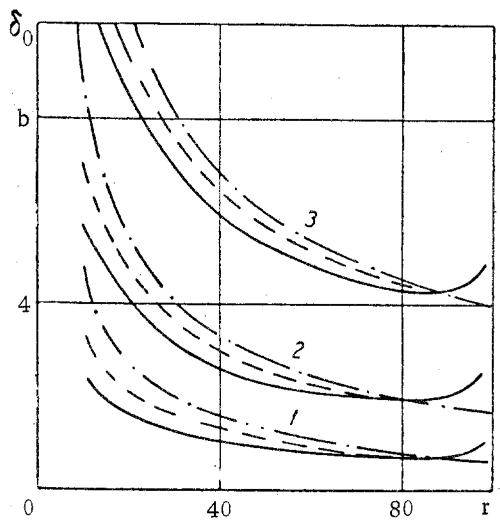

Fig. 2. Dependence of film thickness (δ_0, mm) on radius of surface (r, mm) for a sphere (continuous line), curved diffuser (dot-dash), and a conical surface (broken line): 1) n = 0.5; 2) 1.0; 3) 2.0.

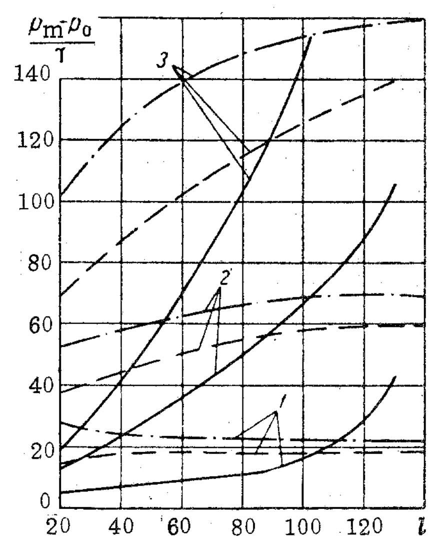

Fig. 3. Dependence of pressure head ($p_m - p_0)/\gamma$, mm, on the length of path l, mm; for notation, see Fig. 2.

For this case we have

$$F_l = \frac{r}{R}\left(\omega^2\sqrt{R^2 - r^2} - g\right),\quad F_\delta = -\frac{1}{R}\left(\omega^2 r^2 + g\sqrt{R^2 - r^2}\right).$$

Figures 2 and 3 show graphs of the dependence of the film thickness δ_0 on the radius, and of the pressure head on the path length of the liquid at various n for a sphere (R = 0.1 m), a cone ($2\beta = 75°$), and a curved diffuser (B = 1, a = = 2/3.

It can be seen from Fig. 2 that the film thickness increases as n increases. When r < 0.85 R, a spherical surface will have the least value of the film thickness, and a curved diffuser the greatest. In the region where r > 0.85 R, a sharp increase of film thickness is observed on the sphere.

It may be seen from Fig. 3 that increase in n leads to increase in pressure head. With increase in the path length traced out by the liquid, the pressure head increases, particularly sharply in the case n = 2 for a spherical surface. The pressure head on a cone and on a curved diffuser varies only slightly along the length of a generator.

In drawing the curves it was further assumed that: $q = 5 \cdot 10^{-5}\ m^3/sec$, $\omega = 100\ sec^{-1}$, $\rho = 980\ N \cdot sec^2/m$, K = = $9.8\ N \cdot sec/m^2$.

NOTATION

τ—shear stress; $\dot{\gamma}$—rate of shear strain; K and n—rheological constants of the liquid; D_σ—stress deviator; D_ε—strain rate deviator; μ^*_{eff}—effective viscosity in the general flow case; μ_{eff}—effective viscosity for one-dimensional flow; T—intensity of shear stresses; E—intensity of rates of shear strain; p_{11}, p_{22}, p_{33}, p_{12}, p_{13}, p_{23}—stress tensor components; v_1, v_2, v_3—projected velocities on the axes q_1, q_2, q_3, respectively; H_1, H_2, H_3—Lamé coefficients; T°—temperature; c_p—specific heat; λ—thermal conductivity; A—thermal equivalent of work; D—function expressing dissipation of mechanical energy; l, φ, δ—respectively, generator of the curved surface, longitude, and distance of a particle of liquid M from the surface; r—distance from surface to axis of rotation; α—angle between the tangent to the curve l at the point M' and the abscissa axis; 2β—cone apex angle; x', y', z'—coordinates of particle M in a rectangular system; x, y, z—coordinates of the point M' lying on the surface; v_l—meridional velocity; F_l, F_δ—projections of mass forces in directions l and δ, respectively; p and p_0—flow pressure and atmospheric pressure; δ_0—film thickness; ρ—density; q—flow of liquid per second; R—sphere radius; ω—angular velocity of rotation of surface; g—force of gravity.

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Kirov Institute of Chemical Engineering,
Kazan